

## CUBOCTAHEDRON AS A POTENTIAL EVIDENCE OF THE “CULTURAL BRIDGE” BETWEEN KYOTO AND KAYSERI

Hakan Hisarligil

*Meliksah University, Turkey*

**Keywords:** Vector equilibrium, cuboctahedron, R.B. Fuller, Kayseri, Kyoto.

### 1. Introduction

There are several abstract geometrical forms ascribed to have a symbolic meaning in various cultures through history. Among these cuboctahedron, one of 13 Archimedean solids has been an interest of art and religion in Japan in the past [1]. Recently the existence of cuboctahedron discovered in other cities and countries on the “Silk Road” also leads to question whether they represent similar content as well [2]. However the examples here in this short text are to be limited to the ones discovered in downtown Kayseri belonging to the 13<sup>th</sup> century. Thus, following the trace of cuboctahedron, we will try to demonstrate the potential evidences of cultural affiliation between two cities of remote geographies through this study.

### 2. A Brief Outlook to Cuboctahedron

#### 2.1. Cuboctahedron "By Any Other Name"

The cuboctahedron is named thusly because it is simply an intersection of a cube and an octahedron, as represented in the "Crystal" by M.C. Escher in 1947 [3]. Associating the episode "By Any Other Name" of *Star Trek* TV series (Fig. 1), where aliens seize the Enterprise by transforming crew members into inanimate cuboctahedron [4], it is attributed numerous names in geometry such as *triangular gyrobicupola*, *cantellated tetrahedron*, *rectified cube* and *heptaparallelhedron*. Buckminster Fuller applied the name "Dymaxion" to this shape along with “*Vector Equilibrium*” [5]. Further, in “sacred geometry” this shape is known as “Heart Sphere” or “Terra Prana Sphere”, since it represents both the earth (cube), and air/prana (octahedron), implying the perfect synthesis for just about everything [6].

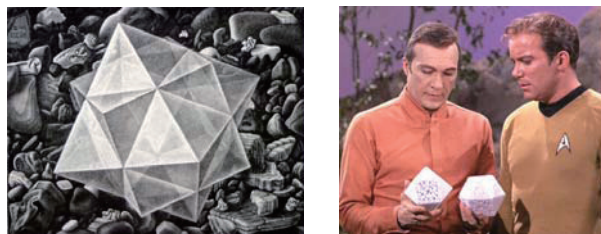


Fig. 1: Left: Mezzotint "Crystal" by M.C. Escher, 1947, Right: "By Any Other Name", *Star Trek*

#### 2.2. Cuboctahedron through History

The first appearance of cuboctahedron is in the book titled as “Archimedean Solids”, which Pappus of Alexandria lists solids and attributes to Archimedes, though Archimedes makes no

mention of these solids in any of his works [7]. Long after, it reappears in Luca Pacioli's book "De divina proportione" written around 1497 where all figures are drawn by Leonardo da Vinci [8]. Johannes Kepler (1571-1630) was the next to write about the Archimedean solids collectively in his book "Harmonices Mundi" [9]. In 1950, Dr. Derald Langham, agricultural geneticist and author of "Circle Gardening", developed the "Genesa Concept" [10]. He believed that Genesa crystal is a sacred geometric shape called cuboctahedron that uniquely contains within it all of the five platonic solids that are the building blocks for all organic life. Cuboctahedron is at the center of Buckminster Fuller's philosophy. Fuller calls this shape the "Vector Equilibrium" meaning as the dynamic balance of tensional cosmic forces, since, unlike Cartesian coordinate system, it can strikingly be developed around one nuclear sphere [11]. Equilibrium of this kind is also called 'isotropic vector matrix' as an omnidirectional closest packing around a nucleus about which omnidirectional concentric closest packing of equal spheres about form series of vector equilibria of progressively higher frequencies (Fig. 2). Following Fuller, the physicist Nassim Hameiri in his unified field theory suggests that the structure of space-time is a cuboctahedral "vector equilibrium". According to his theory, the structure can be seen in the close-packed hexagonal cells of honeycombs and bubbles, boiling water, and the storms on gas giants [12].



Fig. 2: Cuboctahedron through history Left: Pappus, Middle: Leonardo da Vinci, Right: Fuller

### 3. Cuboctahedron in Kyoto and in Kayseri

#### 3.1. Cuboctahedron in Kyoto

Cuboctahedron seems to have had a special meaning for religious people in Japan. It is still open to discussion that in the past the most revered solid symbol was not the cuboctahedron but the *Hoju* gem, a chest-nut shaped solid. It is known that it had been widely used as decorations in furniture and buildings in Japan in the past, since cuboctahedral decorations can easily be made and practically used. Lamps called *Kiriko*, in the shape of cuboctahedra were appeared as a lantern in pictures as early as 13<sup>th</sup> century and they are still used today in certain religious ceremonies in memory of the dead (Fig. 3) [13]. Besides, numerous examples of *Hoju* may found in the sanctuary of a shrine or a temple. In particular, a big *Hoju* usually is put on the top of the *Gorinto pagoda*, a five storied small pagoda, which is the most typical monument for the Buddhist in Japan (Fig. 3). This pagoda is made of five blocks which symbolizes the earth (a cube), water (a sphere), fire (a square pyramid or sometimes a tetrahedron), air (a hemisphere), and the universe (a Hoju), from bottom to top [14]. There is an opinion that such construction might be derived from Plato's book entitled "Timaeus". If it is so, the Hoju represents the regular dodecahedral universe of Plato (Fig. 3).



Fig. 3: Left: Kiriko lanterns in a Bon ceremony Middle: Pagoda Right: "Hoju" gem

Symbolizing the God, it also forms the decoration or monuments such as *pagoda*, the main hall of a temple or shrine (Fig. 4) or at the top of sacred buildings (Fig. 5).



Fig. 4: A cuboctahedral sacred offering completed in the middle of the 17<sup>th</sup> century, tomb of Tokugawa Ieyasu, Nikko



Fig. 5: Cuboctahedral top decorations on Imperial monuments (tea house) in Shugakuin Imperial Palace, Kyoto

Hargittai has also reported that there are some cuboctahedral top decorations on garden lantern (“Toro” in Japanese) in Shugakuin Imperial Palace in Kyoto (Fig. 6) [15].



Fig. 6: Cuboctahedron examples on top of a garden lantern in the Shugakuin Imperial Villa in Kyoto

### 3.2. Cuboctahedron in Kayseri

The first Turkish contact with the political power in Islam was in the 11th century at the hands of the Seljuks. Among the many Turkish States and cultures formed throughout history, the Seljuks have a very significant place and their art represents an important milestone within Turkish art [16]. Involving common specific features of Islamic art and architecture in general, numerous works of Seljuk Turks in Anatolia represent abstract geometrical dimension of the Muslim World particularly. According to Rabah Saoud, the artists of that era used and developed geometrical art for two main reasons: “*The first reason is that it provided an alternative to the prohibited depiction of living creatures. Abstract geometrical forms were particularly favored in mosques because they encourage spiritual contemplation, in contrast to portrayals of living creatures, which divert attention to the desires of creatures rather than the will of God. Thus geometry became central to the art of the Muslim World, allowing artists to free their imagination and creativity. A new form of art, based wholly on mathematical shapes and forms, such as circles, squares and triangles, emerged. The second reason for*

*the evolution of geometrical art was the sophistication and popularity of the science of geometry in the Muslim world. They also show that early Muslim craftsmen developed theoretical rules for the use of aesthetic geometry, denying the claims of some Orientalists that Islamic geometrical art was developed by accident (e.g. H. Saladin 1899)* [17]. Indeed, the recently discovered Topkapi Scrolls by Gulru Necipoglu [18], dating from the 15th century, illustrate the systematic use of geometry by Muslim artists and architects. Further, showing how mathematicians instructed artisans, Alpay Ozdural provides us with an insight into the explicit collaboration between mathematicians and artisans in the Muslim World [19]. Remarkable works on Seljuk art (Makovicky, 1992; Alexander, 1993; Lu & Steinhardt, 2007) [20] also conclude and suggest that artisans of the era had an intuitive understanding of highly complex mathematical problems. Likewise, the recent discovery of numerous examples of cuboctahedron of 13<sup>th</sup> century in Kayseri in 2009 [21], also indicates such a complex geometrical content of the art of the era. In fact, cuboctahedron is a quite common figure in iron window grills of the buildings of both Seljuk and Ottoman era in Turkey. Further, the base of almost all kumbets also implies almost an upper part of a cuboctahedron on the ground surface (Fig. 7). However, there has been found no written record indicating their existence and possible meaning so far.



Fig. 7: Cuboctahedron on kumbets and iron window grills

Hence, demonstrating the examples discovered in Kayseri, numerous possible meaning will be attributed to cuboctahedron under the guidance of extensive explanations on cuboctahedron in “Synergetics” by R.B. Fuller who is putting cuboctahedron at the top of the cosmic hierarchy as center of creation. The first one is if the cuboctahedron replaced by some other forms of capital of colonnade such as ‘tree of life’ and ‘muqarnas’ represent cosmos, humanity or the entire creation with the creator veiled (Fig. 8). Second whether the niche in the form of cube enclosing almost all examples represents the location of eternal energy as described by the God in the “Al-Noor” (The Light) verse of holy Quran [22].



Fig. 8: Left: Examples of cuboctahedron enclosed by a niche in the form of cube Right: examples of ‘tree of life’ and muqarnas replacing cuboctahedron

Bearing in mind that octahedron represents “air” and cube represents “earth” for Plato, cuboctahedron placed in the middle of the arches of main portals associates the balance as the worldly creatures between ‘earth and sky’ (Fig. 9) [23]. If so, beside this vertical balance, the animal figures placed on the cuboctahedron, a bird on the left and a lion on the right, at the main portal of Karatay Caravanserai, would virtually represent the Yin-Yang as the

'harmony of opposites' common both to Eastern and to Western philosophy since ancient times (Fig. 9) [24].

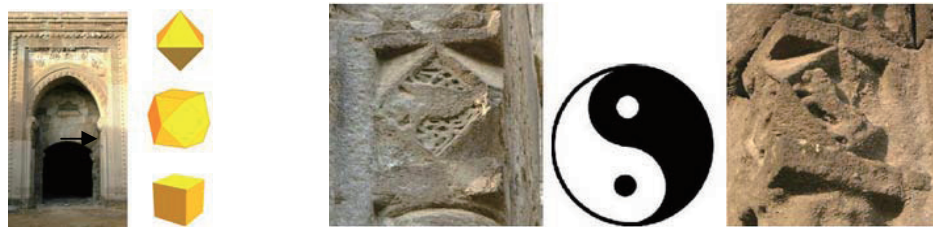


Fig. 9: Left: Location of cuboctahedron at main portals, Right: Cuboctahedron with animal figures at Karatay Caravanserai

Further, the octahedron discovered at the main portal of Kilij Aslan Mosque leads us to think whether the artisans of 13th century have already had the idea of “jitterbug” representing the phase transformation between cuboctahedron and octahedron as envisioned by Fuller at the end of 20th century (Fig. 10) [25].



Fig. 10: “Jitterbug” representing the phase transformation between cuboctahedron and octahedron

#### 4. Discussion and Conclusion

After having observed the examples presented above, any one clearly trace that two cities of remote geographies, Kayseri and Kyoto, had explicitly shared something common in the past: Cuboctahedron. Regardless the differences in terms of size, position and material etc. between the examples, they are somehow a form of a secret manifestation of Japanese and Turkish designers practicing for either sacred or practical purpose. The examples of cuboctahedron of both cultures in their unknown meaning and context can be regarded as not only as the evidence of cultural affiliation between two locations but further as the road map of guiding throughout both ends of the “Silk Road”. However, having discovered numerous other examples in other cities and countries, it is obvious that this limited survey is far from covering the subject and geography it deserves. Our hope is that, by drawing the attention of scholarly circles on that relatively new and exciting content, the cuboctahedron will lead a series of colorful researches bridging the East and the West as it was in the past.

#### References

- [1] Hisarligil, H., & Bolak-Hisarligil, B. Back to the Future: “Cuboctahedron” Revisited. *Design Principles and Practices: An International Journal*, Vol. 3, No.4, pp.109-124, 2009.
- [2] *The exhibition: M. C. Escher: Cycle, Spirals, and Snakes.*  
[http://www.artistsmarket.com/files/MCE\\_10.0\\_CSS\\_Gallery.pdf](http://www.artistsmarket.com/files/MCE_10.0_CSS_Gallery.pdf) (22/03/2012)

- [3] *By any other name*. [http://en.wikipedia.org/wiki/By\\_Any\\_Other\\_Name](http://en.wikipedia.org/wiki/By_Any_Other_Name) (22/03/2012)
- [4] *Cubeoctahedron*. <http://en.wikipedia.org/wiki/Cubeoctahedron> (22/03/2012)
- [5] *Sacred geometry*. [www.zakairan.com/.../SacredGeometry.htm](http://www.zakairan.com/.../SacredGeometry.htm) (22/03/2012)
- [6] Pappus of Alexandria, *Archimedean Solids*. <http://www.math.nyu.edu/~crrres/Archimedes/Solids/Pappus.html> (22/03/2012)
- [7] Pacioli, L. *De Divina Proportione*. Silvana, 1982. (In Italian)
- [8] Kepler, J. *Harmonices Mundi*. [http://en.wikipedia.org/wiki/Johannes\\_Kepler#Harmonices\\_Mundi](http://en.wikipedia.org/wiki/Johannes_Kepler#Harmonices_Mundi) (22/03/2012)
- [9] Langham, D. *Genesa Crystals*. <http://www.genesa.org/> (22/03/2012)
- [10] Fuller, R.B., Applewhite, E.J., & Loeb, A.L., *Synergetics: Explorations in the Geometry of Thinking*, Edition: 2, Macmillan, 1975.
- [11] Haramain, N. *Harmonics study postage*. [http://www.zazzle.com/harmonics\\_study\\_postage-172904018863995877](http://www.zazzle.com/harmonics_study_postage-172904018863995877) (22/03/2012)
- [12] Miyazaki, K. The Cuboctahedron in the Past of Japan. *Mathematical Intelligencer*, Vol.15, No.3, pp.54-55, 1993.
- [13] Hargittai, I. Imperial Cuboctahedron. *Mathematical Intelligencer*, Vol.15, No.1, pp. 58–59, 1993, Miyazaki, K. The Cuboctahedron in the Past of Japan. *Mathematical Intelligencer*, Vol.15, No.3, pp.54-55, 1993.
- [14] Ibid.
- [15] Ibid.
- [16] Saoud, R. *Muslim Architecture under Seljuk Patronage (1038-1327)*. Foundation for Science, Technology, Civilisation Publication No: 4041, Manchester, 2003.
- [17] Saoud, R. *Introduction to Islamic Art*. <http://muslimheritage.com/topics/default.cfm?ArticleID=1190>, (04.06.2012)
- [18] Necipoglu, G. *The Topkapi Scroll. Geometry and Ornament in Islamic Architecture*. The Getty Center for the History of Arts and the Humanities, Santa Monica CA, 1995.
- [19] Ozdural, A. The Use of Cubic Equations in Islamic Art and Architecture. In *Nexus IV: Architecture and Mathematics*, ed. J. F. Rodrigues and K. Williams. Kim Williams Books, Turin, Italy, pp. 165-179, 2002.
- [20] Makovicky, E. 800-year-old Pentagonal Tiling from Maragha, Iran, and the New Varieties of Aperiodic Tiling It Inspired. In *Fivefold Symmetry*, ed. I. Hargittai. World Scientific Publishing, Singapore, pp. 67-86, 1992. Alexander, C. *A Foreshadowing of 21st Century Art: The Color and Geometry of Very Early Turkish Carpets*. Oxford University Press, New York, 1993. Lu, P.J., & Steinhardt, P.J. Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture, *Science*, No.315, pp.1106-1110, February 2007.
- [21] Hisarligil, H., & Bolak-Hisarligil, B. Back to the Future: “Cuboctahedron” Revisited. *Design Principles and Practices: An International Journal*, Vol. 3, No.4, pp.109-124, 2009.
- [22] Chapter 24, *Al-Noor*. <http://www.submission.info/servlet/qtbrowse?pickthall=true&yusufali=true&arabic=true&chapter=24&verseBegin=30&verseEnd=31> (22/03/2012)
- [23] Hisarligil, H., & Bolak-Hisarligil, B. Back to the Future: “Cuboctahedron” Revisited. *Design Principles and Practices: An International Journal*, Vol. 3, No.4, pp.109-124, 2009.
- [24] Ibid.
- [25] Ibid.