APPLICATION OF LOG-AESTHETIC CURVES TO THE EAVES OF A WOODEN HOUSE

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Keywords: LAC, natural object, gable roof, straight line, curvature, divergent type, cutting line, full scale drawing, fascia, construction accuracy

Introduction

In most contemporary buildings, shapes have straight lines and surfaces are flat. In buildings with curved surfaces, simple forms, including arcs and cylinders, that are easily described in Euclidean geometry are typically used. On the other hand, for most natural objects, shapes and surfaces are curved. These curves have complex forms that are more difficult to describe in Euclidean geometry. Globally, including areas along the Silk Road, buildings formerly included such natural shapes and surfaces. As time has passed, however, many of these have been replaced with simpler forms made up with straight lines and flat surfaces.
While we naturally design using straight lines and flat surfaces, at the same time, we also find beauty in natural curved shapes and surfaces, which are difficult for us to determine and use in architectural design. This seems to be one of the reasons why they are hard to develop in build-to-order buildings, especially houses, as opposed to industrial products, such as cars.
In this research, I experimented with log-aesthetic curves (LACs) [1][2][3], which share characteristics with the curves in natural objects, as eaves for a gabled wooden house as an example of architectural design. My aim is to reveal findings necessary to design houses using such curves.

Methods and Results

(I) SETTING CONDITIONS

The floor plans of the house are described in Fig. 1. For the purpose of this research, I studied the upstairs eaves of the gable roof. I set the conditions of the eaves as shown in Fig. 2. Roof W's pitch is 5:10 (26.57°) and Roof E's pitch is 2.5:10 (14.04°). Each of the three curves, C_W, C_E, and C_S, was selected to connect smoothly with its adjacent straight line. No gutters are mounted on the roofs because of the proximity of deciduous trees.

- C_W connects points W_0 and W_1. The slopes of the tangents at W_0 and W_1 are as shown in Fig. 2. The curvature of C_W at W_0 is 0.
- C_E connects points E_0 and E_1. The slopes of the tangents at E_0 and E_1 are as shown in Fig. 2. The curvature of C_E at E_0 is 0.
- C_S passes through point S_0 and the curvature of C_S at S_0 is 0. C_S is determined to coincide with a part of the curve geometrically similar to right and left reversed curve of C_E. The reason for using a curve geometrically similar to the reversed curve will be explained later.

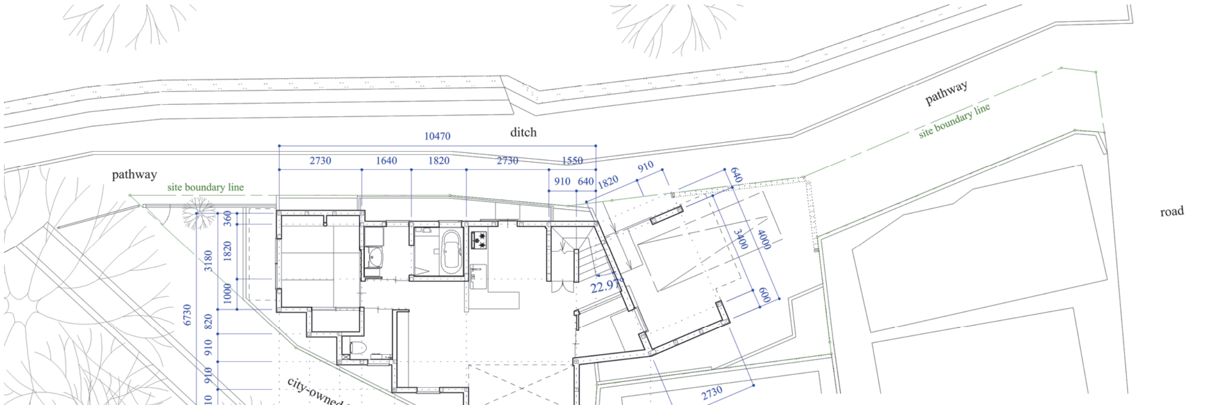
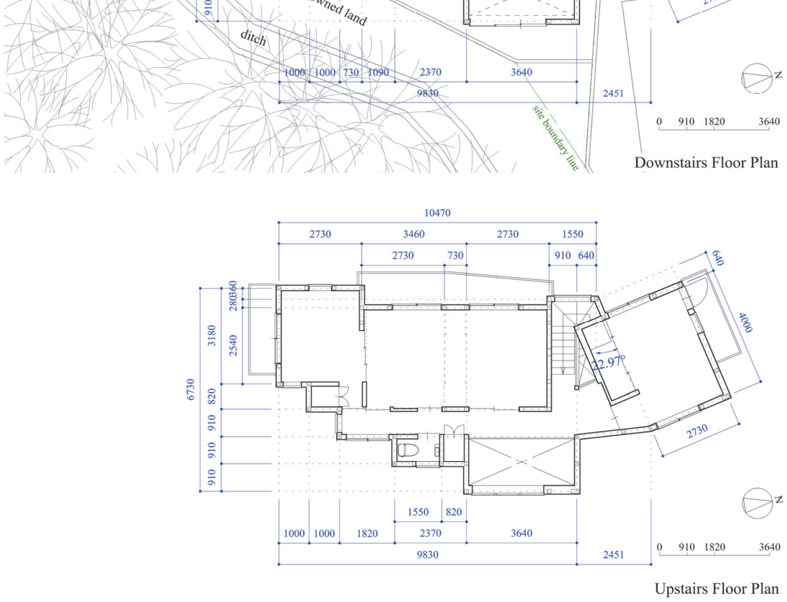

Fig. 1: Site and floor plans S=1:250

(II) DETERMINING CURVES

I used log-aesthetic curves (LACs) for determining the three curves because many beautiful curves in natural objects, such as shellfish, butterflies, calabashes, and beetles, craftwork, such as Japanese swords and violins, and industrial products, such as cars, are approximated by LACs [1][2][3].

The LAC satisfies the following basic equation [3]:

$$\log\left(\rho\frac{ds}{d\rho}\right) = \alpha\log\rho + C \quad (1)$$

where ρ is the curvature radius, s is the arc length, and α and C are constants.

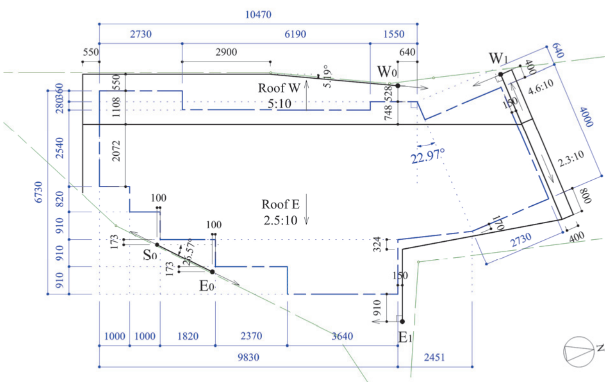

Fig. 2: Conditions of eaves on roof plan S=1:250

When $\alpha \neq 0$, the general equation of the LACs is:

$$\rho^{\alpha} = c_0 s + c_1 \quad (2)$$

where c_0 and c_1 are constants.

When $\alpha = 0$, the general equation of the LACs is:

$$\rho = c_0 e^{c_1 s} \quad (3)$$

The LACs can be classified into divergent $(\alpha < 0)$, constant speed $(\alpha = 0)$, and convergent $(\alpha > 0)$ types [2]. Divergent type LACs with $\alpha = -1$ are referred to as clothoid curves and convergent type LACs with $\alpha = 1$ are referred to as logarithmic spirals [3].

Many curves found in natural objects and craftwork are of the divergent or constant speed type [2]. The divergent type LAC can connect to a straight line smoothly since it can have a point where its curvature is 0. On the other hand, the connecting point of the constant speed type LAC and a straight line always have no curvature continuity since the curve cannot have a point at which its curvature is 0. The convergent type LAC has the same issue [4].

Therefore, I used divergent type LACs to obtain the curves satisfying the conditions described in (I). The LACs were determined not on the roof plan (Fig. 2), but on the plane containing Roofs W or E. The reason for this is that the LACs found in natural objects, craftwork, and industrial products are coplanar, and a curve obtained by projecting the LAC on a plane not parallel to it does not usually satisfy the LACs general equation.

I drew LAC C_W connecting W_0 and W_1 on Roof W using LAC Plugin [5], and set α so that the curvature of C_W at W_0 was 0. As a result, α of C_W became -2.480. In the same way, α of C_E on Roof E became -4.247 (Fig. 3).

It was difficult to determine the α of LAC C_S uniquely after setting its both endpoints because a portion of the curve got too close to or crossed over the site boundary line. Therefore, I first fixed α of C_S to be the same as α of C_E since C_S *and* C_E are apparent at the same time. The C_s was reduced from right and left reversed curve of C_E to be at least 40 mm away from the site boundary line because the reversed curve crossed over the line. The reduction ratio was 0.622 (Fig. 3).

Fig. 4 shows the roof plan and the Fig. 5 shows the elevations.

(III) PRINTING FULL SCALE DRAWING

The allocation of roofboards is shown in Fig. 6. I printed the full scale drawings, including the determined curves and the allocation, on paper rolls. The cutting lines of the roofboards from 20 mm inside of the curves were also printed on the drawing, since the roof is 20 mm larger than the cutting lines.

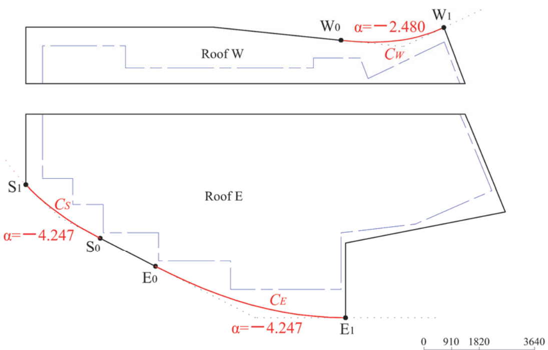

Fig. 3: Determined LAC C_w on the plane containing Roof W and LACs C_E and C_S on the plane containing Roof E S=1:250

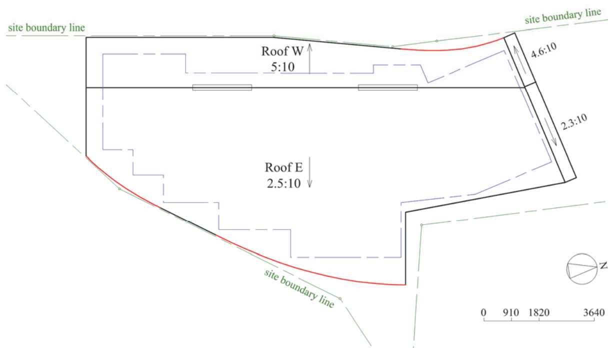

Fig. 4: Roof plan including three curves S=1:250

Fig. 5: West and east elevations S=1:250

(IV) CONSTRUCTION OF ROOFS

The full scale drawings were placed on the roofboards (Fig. 7). The roofboards were cut on the cutting lines as per the drawings. They were then roofed with asphalt roofing and Galvalume steel plates, and fasciae were mounted (Figs. 8-11).

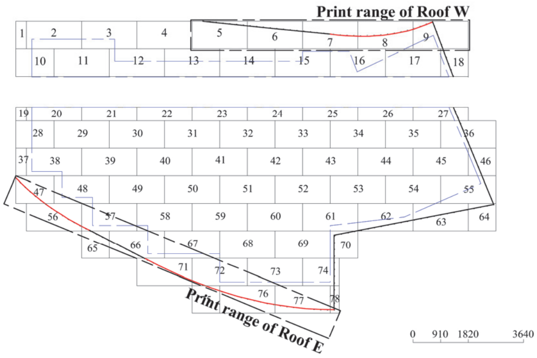

Fig. 6: Allocation of roofboards, cutting lines and print range for full scale drawing S=1:250

Discussion

The curves of eaves can be designed to connect smoothly in a straight line using the divergent type LACs. However, increasing the construction accuracy of the fasciae is not as easy as expected because of its complex curved surface. It would appear that another solution is needed.

Conclusion

This research clarified that the curves of the eaves of a gabled wooden house can be designed and constructed with the help of divergent type LACs sharing characteristics with the curves in natural objects. In the future, we will explore an easier method of increasing the construction accuracy of the fasciae.


Fig. 7: Full scale drawing placed on roofboards

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Fig. 9: Appearance from the north pathway


Fig. 8: Appearance from the northwest side


Fig. 10: Appearance from the east side


Fig. 11: Appearance of the east side